Calculations of models for non-adiabatic processes

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A melting element very often is sheet metal containing parts of reduced cross-section which represent weak areas when current is flowing through. Undoubtedly a melting element therefore is a two-dimensional structure. But yet it is possible to have the process of heating-up the melting element by a short circuit current described very neatly by a simple one-dimensional model. The advantages of such a method are simpler relations, less supporting points for the calculation of the temperature along the melting element and therefore less time for the calculation of the temperature distribution.

The processes of heating-up and melting both combined with heat transport along the metal are demonstrated by simple differential equations. These equations are solved by approximation by means of a small computer and the results are compared with experimental values.

Material constants:

abbreviations , definitions , dimensions , numerical values		Ag	Cu
Ao = resistivity of the conductor at 20 degr. cent.	O*mm^2/mm	.000016	.0000175
A1 = first order temp. coeff. of resistivity	1/K	.00377	.0039
A2 = second order temp. coeff. of resistivity	1/K^2	5.7 E-7	6.7 E-7
A3 = thermal conductivity k at 20 degr. cent.	W/(mm*K)	.428	.385
A4 = first order temp. coeff. of thermal conductivity	1/K	- .00018	- .0002
A5 = thermal capacity c at 20 degr. cent.	W*s/(g*K)	.237	.377
A6 = first order temp. coeff. of thermal capacity	1/K	.00020	.00024
A7 = coeff. of linear thermal expansion	1/K	.000024	.000020
A8 = density d of solid conductor at 20 degr. cent.	g/mm^3	.0105	.00895
A9 = resistivity of liquid conductor at melt. temp.	O/mm^2/mm	.00016	.00022
Bo = melting temp. Ts of the material, degr. cent.	K	960	1083
B1 = latent heat of the conductor material	W*s/g	105	214
W9 = resistivity of solid conductor at melt. temp.	O*mm^2/mm	.0000808	.000103

B4 = density of the liquid conductor and B8 = density of solid conductor at melt. temp.

1.1. Current heat and heat loss along the melting element

The behaviour of a melting element at very short current pulses can be described by a model for adiabatic heating. When the current pulses become longer there will be heat loss during the time the current flows. The heating up will be delayed and the melting time will grow up. If the melting element is a wire of constant cross section maximum melting time and current for adiabatic heating can be determined according to (4). A lower current heat and a delayed melting time mean heat loss by heat conduction along the wire.

These melting times may reach 50 ms for respective wires.

In this essay we will only look at the heat loss by conduction along the melting element. Longer current pulses which lead to additional heat loss into the sand are forbidden.

A lot of melting elements consist of homogenous sheet metal with rows of accurately punched restrictions in it. This results in parts of smaller cross section on the melting element. A current flowing through a small cross section means large heating but little heat conduction. A large cross section at the same current means little heating but remarkable heat conduction and therefore great influence on the melting time.

The instantaneous value of the current can determine whether the current heat exceeds the heat loss by conduction and heats up the melting element or whether the heat loss at small values of the current prevents the temperature rise.

In the following model calculation we limit the melting time to 50 ms. In this period only heat loss along the metal exists. Heat loss to surrounding sand is not yet important.

We construct for the calculation a model that contains simple differential equations and that enables us to solve the formulas by means of a small computer. We only want the solution to be as correct as the result found by the experiment. The calculation will be made stepwise by approximation. At the beginning of the experiment or the calculation we define a temperature portrait of the melting element and we calculate the temperature values changed after a fixed time interval dt.

1.2. Construction of the model of the calculation

1.2.1. The melting element as a one-dimensional model

A high current flowing along a broad melting element will be driven by its dynamic forces to the lateral edge of the element. We will neglect this effect and assume homogenous current density. We imagine the melting element is cut lengthwise in small parallel strips. A current pulse along such a strip will warm up the restrictions on the strip. The material between the restrictions stays at its temperature. Because of the good thermal conductivity of the metal we have constant temperature transverse to the strip, but not constant along the strip. The temperature on that part of the strip without restrictions is constant in all directions.
As a first approximation we obtain a one-dimensional temperature portrait along each parallel strip of the broad melting element. The constant current density across the whole melting element causes the temperature to have the same value transverse to all parallel strips.
By this method we can calculate the temperature portrait on a broad melting element with a one-dimensional model. The formulas are much simpler. We need less supporting points for the temperature. This means less memory capacity and calculating time and results in the applicability of a small personal computer.

1.2.2. The restrictions on a strip

In many cases a melting element contains several rows of restrictions normally symmetrical in respect to the midst of the restriction. This limitation does not alter the method of calculation. Current heat will produce the highest value of the temperature in the midst M1 of the restriction. In the midst M2 of two rows of restrictions current heat will be minimal. The heat flows from the midst M1 to the midst M2 between two rows of restrictions. The effect of heat conduction is symmetrical in respect to the midst M1 of the restriction. Therefore it is sufficient to select for the model a region M1-M2 on one small strip of the melting element. The calculating model enables us to calculate the temperature values on the region M1-M2 as a function of the instantaneous time.
Production of the heat by the current and heat conduction along the strip are simultaneous. Because we want to have a very simple model we assume heat production and heat conduction to be two effects at two different moments following close to each other.
Then we can calculate both effects separatedly and simply add the results. The heating-up can be treated as adiabatic heating and it is possible to calculate with known simple formulas.

1.2.3. The model of the chain of pearls

For the calculation of the heating-up of the melting element the current density and the time after the commencement of the current must be known. The heat conduction will be calculated with the values of the cross section and the distance of the transportation of the heat, the temperature difference at the ends of the distance and the fixed time interval. Because the shape of the resctrictions shall not be significant for the solution of the differential equations we must find a solution independent of the shape of the restrictions.
This will be gained by deviding the region M1-M2 of the melting element into several small intercepts m. Each intercept is described by its real length, by a mean value of the cross section for the calculation of the current heat and by a realistic cross section for the heat conduction.
We assume furthermore the mass to be heated up will be concentrated in one point in the midst of each intercept. With the above mentioned model the temperature deviation and the temperature of each mass point will be calculated.
Normally the cross section varies in each intercept. As to the heat conduction we must define two different values for the cross section at both ends of the intercept. The distance for the heat to flow to within the time interval dt is described by the distance x of the mass points and by an active cross section between the two mass points.
This delivers us a model like a chain of pearls. It shows us a stepped temperature curve.
We divide one restriction from its midst M1 to its end M3 into several intercepts of different length and we have the intercept from M3 to the end M2 as the last intercept with only one temperature value in each midst point. The number of intercepts depends on the shape of the restrictions. When the cross section changes very much with the distance of the midst M1 of the restriction it is helpful to have ten intercepts. When the cross section is nearly constant six intercepts will result in a sufficient correct value. A greater number of intercepts does not deliver very much better results but will need more time for calculation. Because a higher temperature of the material delivers a higher current heat it is suggested to have the length of the intercepts adapted to its cross section.

1.3. Current heat in the solid material

The following calculation is valid for each intercept m of the region M1-M2. R(T,m) is the resistance of the intercept at the beginning of the time interval, dt the length of the time interval for the calculation of the current heat dQa and i(t) the instantaneous value of the current in the middle of the time interval dt. When we refer the literature (4) we find a maximum time tm without remarkable heat conduction for each length x.

1.3.1. Ag : $tm = 0.5*x^2$ Cu : $tm = 0.8*x^2$ (tm in ms ; x in mm)

In general a current i(t) produces current heat dQa within the time interval dt

1.3.2. $dQa = R(T)*i^2(t)*dt$

We obtain the resistance R(T,m) of the selected intercept m when we know the length x of the intercept, an average cross section q and the resistivity Ro. These three values depend on the temperature and on the measures of the intercept m. The resistance is

1.3.3. $R(T,m) = Ro*x/q = Ro(T) * x(T) / q(T)$

As shown before in (4) we express each value as a product of a material constant independant on the temperature and of the beginning of a power series of the temperature T. We collect all values depending on the temperature T to a temperature function f4(T).

1.3.4. $f4(T) = (1 - A1*To + (A1-2*A2*To)*T + A2*T^2) / (1+A7*T)$

After a short time the function f4(T) reaches a different value for each intercept in the same moment. Therefore the resistance depending on the temperature of the intercept m will be

1.3.5. $R(T,m) = (Ao*xo/qo) * f4(T)$

Ao , xo , qo are the values of the intercept m at a temperature of 20 degrees centigrade. We write the current i(t) as a product of a constant value D and a time function f(t). After n time intervals dt we take the value of the current i(t) in the moment t = (n-1/2)*dt and the temperature values at the beginning of the time interval dt and obtain the current heat dQa(m) of the intercept m produced in the time interval dt.

1.3.6. $dQa(m) = (Ao*xo*/qo) * f4(T,m) * D^2 * f^2(t) * dt$

Because we divide the restriction M1-M3 of the segment M1-M2 into several intercepts we will have for each intercept a differential equation ananlogous to equ. 1.3.6.
In general this cross section is not identical to the cross section for the heat conductivity from one intercept to the next one. The last intercept M2-M3 of the section M1-M2 has got a constant scross section.
Because the current through all intercepts of a section M1-M2 has the same value but the cross section and the energy necessary for the melting of the intercept normally not we calculate with the value of the current i(t).
At the beginning of the current all intercepts shall have the same temperature value To.
In the first time interval dt the current i(t) produces an amount of heat dQa(m) in the intercept m.

1.3.7. $dQa(m) = (Ao*xo/qo) * f4(To,m) * D^2 * f^2(dt-dt/2) * dt$

After a period of n time intervals dt the intercept m has gained the temperature Tm(m).
The current i(t) produces an amount of heat dQa(m) in the intercept m in the next interval dt.

1.3.8. $dQa(m) = (Ao*xo/qo) * f4(Tm,m) * D^2 * f^2(n*dt-dt/2) * dt$

1.4. The heat conducted away in the solid material

We begin with a simple equation for the heat conduction. The heat dQk(m) which flows from one mass point m of the chain of pearls to the next one is given by the distance x of the two mass points, by an active cross section qk, by the thermal conductivity k of the material of the melting element, by the temperature difference dT of the two mass points and by the length of the fixed time interval dt.

1.4.1. $dQk(m) = k * qk * dT * dt / x$

Because we want the time interval dt to be respectively small according to chapter 1.3. the temperature values of the two mass points alter only a little. Therefore we can assume the temperature difference dT in equ. 1.4.1. as a constant value for each single step.
Because we know that we calculate especially time intervals dt with current heat the model will work. We calculate the amount of heat dQk(m) that flows from one mass point m to the next one. The material values k, qk and x depend on the temperature. The values qk and x depend on the shape of the restriction too. That means they depend on the intercept m.
Once more we write the material constants at the temperature T as a product of values at 20 degrees centigrade and a function of the temperature T(m) of the intercept m .
Hence we obtain the heat dQk(m) which flows from the mass point m to the mass point m+1 within the time interval dt.

1.4.2. $dQk(m) = (A3*qo/xo) * (1+(A4+A7)*T(m)) * dt * (T(m) - T(m+1))$

The active cross section qo is a real value between the two mass points m and m+1 and depends on the shape of each intercept.
At the commencement of the current all intercepts have the same temperature. The heat conducted away therefore is zero. When the current gains a measurable value temperature differences are produced and heat conduction will occur.

1.5. The current heat produced during the transition solid/liquid

When a material is heated up to the melting temperature Ts the temperature rests constant while the current still produces current heat. The resistivity changes during the transition from the value W9 for the solid state to the value A9 for the liquid material. Is the heat necessary to make the material fluid produced by the current then the dynamic forces of the current can interrupt the melting element. The process of the melting is finished. We obtain the the amount of heat Pm necessary for the transition in each intercept when we multiply the latent heat Bl with the mass of the intercept m which depends not on the temperature.

1.5.1. $Pm = Bl * A8 * qo * xo$

A simple relation between the resistivity Ro and the amount of heat Q(m) left in the mass of the intercept m is a linear function. Thus we write the resistivity of the intercept m as a product of a function fl(Q) and the resistivity W9 of the solid material.
The amount of heat added up in the intercept m depends on the current heat produced at the melting temperature Ts and on the heat loss that means on the dimensions of the intercept. The function fl(Q,m) yields different values for each intercept in the same moment.

1.5.2. $fl(Q,m) = (Pm + Q(m)*(A9-W9)/W9) / (Pm + Q(m)*(1-B4/B8)/3)$

fl(Q,m) is of no dimension. In (4) the function fl(Q) is explained in details.
When the intercept m gains the melting temperature Ts the length shall be x(Ts) and the cross section q(Ts). At this moment the value of Q(m) is zero.
The resistance R(Ts,m) of the intercept during the transition solid/liquid then is given by

1.5.3. $R(Ts,m) = W9 * fl(Q,m) * x(Ts) / q(Ts)$

After n time intervals dt we take the value of the current i(t) in the moment t=(n-1/2)*dt and the value of the added-up heat Q(m) at the beginning of the time interval dt and obtain the current heat dQa(m) of the intercept m produced within the time interval dt.

1.5.4. $dQa(m) = W9 * fl(Q,m) * (xo/qo/(1+(Ts-To)*A7)) * D^2 * f^2(t) * dt$

1.6. The heat conducted away during the transition solid/liquid

The calculation of the model for adiabatic heating showed that about 90 % of the value of the melting integral were needed to heat the material up to the melting temperature Ts. The remaining 10 % could do the transition from solid to liquid.
An error of 50 % at the calculation of the transition integral because of an inaccurate model then yields an error of 5 % at the melting integral of the melting element.
The heat conduction during the transition solid/liquid results in a higher value of the heat produced by the current at the melting temperature Ts. The portion of the transition energy may increase up to 50 % of the melting integral. An error in the model for the calculation of non adiabatic heating will influence very much the melting integral of the melting element at longer melting times.
During the transition solid/liquid of the material the temperature of the material remains at the melting temperature Ts. Are the temperature values of two intercepts equal to the value Ts then the temperature difference between these two intercepts is zero. The use of formula 1.4.1. results in no heat conduction.
Arrive these two intercepts at the melting temperature at different moments then the transition solid/liquid normally is not in the same state. The added-up heat of transition in the two intercepts is different and we will have heat conduction between the intercepts. It is necessary to find a method to calculate heat conduction during the transition from solid to liquid. As a first approximation we assume the active force of heat conduction will be the difference of the added-up energy Q in both intercepts.
Hence we yield the heat dQk(m) conducted away from the intercept m within the time interval dt analogous to equ. 1.4.1.

1.6.1. $dQk(m) = k * q(Ts) * (Q(m)-Q(m+1)) * dt / x(Ts)$

The conductivity k of the material changes during the transition solid to liquid. It is difficult to find numerical values in the literature. Hence we assume the conductivity k to be constant during the transition solid to liquid. Comparing of calculated and measured values will show the applicability of this simplification.
When a heat dQ is added to a mass m with the thermal capacity c then the temperature T(m) will change by a value dT according to the relation

1.6.2. $dQ = m * c * dT$

Is the material heated up to the melting temperature Ts before no additional temperature rise will occur. Yet we are not forbidden to define a virtuel inner temperature T(m) for the material. Equ. 1.6.2. gives us the value dT of the change of this inner temperature.
At the beginning of a time interval dt an amount Q(m) of heat of transition is stored in the intercept m and therefore a value of its inner temperature exists. When we add a virtuel temperature difference dT then we obtain a changed virtuel temperature T(m) of the intercept m. This is valid for each intercept.

1.6.3. $T(m) = Q(m) / (m*c) + Ts$

When in equ. 1.4.2. the real temperature difference is replaced by the difference of the virtuel temperatures and the temperature value T(m) by the value Ts then we can calculate the heat dQk(m) which is conducted away from the intercept m to the next one m+1 .

1.6.4. dQk(m) = (A3*qo/xo) * (1+(A4+A7)*Ts) * (T(m)-T(m+1)) * dt

The amount of heat conducted away from intercept m is conducted to and added to the next intercept m+1.
Is intercept m already heated up to the melting temperature Ts but not yet intercept m+1 then it is possible too to calculate the heat conducted away from the intercept m by means of equ. 1.6.4. Then the value T(m) means the virtuel temperature of the intercept m and the value T(m+1) means the real temperature of the intercept m+1 .

1.7. Calculation of the temperature values

Current heat and heat conduction in the melting element define the amounts of energy dQ(m) left in each intercept within the time interval dt. For temperature values below the melting temperature Ts exist other formulas than for temperature values at the melting temperature. When we add the current heat dQa(m), the heat dQk(m-1) conducted to and the heat dQk(m) conducted away from the intercept m we obtain the energy dQ(m) stored in the intercept m within the time interval dt.

1.7.1. dQ(m) = dQa(m) + dQk(m-1) - dQk(m)

When the energy dQ(m) is added to a mass m of the thermal capacity c the temperature of the mass changes by the value dT(m). We obtain

1.7.2. dT(m) = dQ(m) / (m*c) / (1+A6*T)

The thermal capacity c depends on the temperature T, the mass m does not.
Is the intercept m heated up to the temperature T1(m) then after a time interval dt the temperature of the intercept m has changed to

1.7.3. T2(m) = T1(m) + dT(m)

The first calculation round yields the temperature values of all intercepts after the first time interval dt by means of the above mentioned formulas. The same formulas used with the calculated temperature values yield the second series of temperature values of all intercepts. When we go on this way we obtain in every moment the temperature portrait of the chain of pearls which represents the melting element.

1.8. Some details of the programm

The program is written in basic and was developped to run on a cbm 4032 basic computer of 32 KB RAM and 8-bit cpu 6502. Later on the program was tested on a cbm 610 basic computer of 128 KB RAM, 8-bit cpu and 80 columns screen.
This program is made for rising currents only. With little additional software it is possible to calculate temperature values too when the current decreases. Then the run time of the programm will encrease by about 30 %.
When the current passes a maximum before the intercept melts then the heat loss will exceed the current heat and the error of the result increases by using this simple model.
The run time is valid for a program that calculates one type of restrictions only. When there are two or more restrictions of different shape on the melting element all different restrictions had to be calculated under the very same conditions. The restriction which needs the least current heat starts to interrupt the melting element. The total run time of the program is the sum of all single run times of each restriction.
The computer displays all temperature values, the instantaneous time and the instantaneous run time. Temperature values above the melting temperature Ts are displayed as Ts-values. This makes the program comfortable but slow.When we divide the section M1-M3 of a restriction into ten parts of equal length the run time is about 34 minutes for a real melting time of 6 ms. To make the program faster it needs to eliminate the basic lines for the display of the instantaneous values. The run time then decreases to 30 minutes.

1.9. Discussion of the results

It is most important to have accurate numerical values of the material of the melting element. Then we can ask the program for the instantaneous values of time and temperature and for the i^2t-values in every moment.
We find the fact that the values of the melting integral by experiment are lower than the calculated values. The lowest limits are delivered by calculation of adiabatic heating.
The difference between the results of calculation and experiment may be caused by incorrect numerical values of the material, by inhomogenous current density in the melting element and by the fact that until now the tolerances of the values in the experiment were not measured as accurate as it would be necessary today. For silver material only a few values existed. For copper material we had a lot of values and we found a result by calculation as accurate as by experiment.

Table 1.9.1.

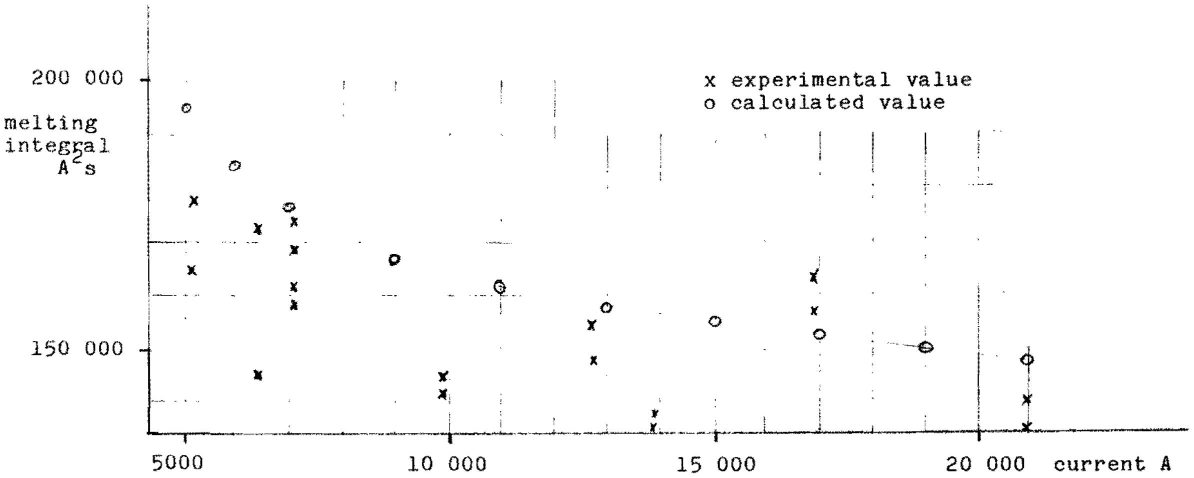
material	silver	copper		
melt.integr. at	132.5	168.7	A^2s	experiment
virt.time 4 ms	162.7	179.3	A^2s	calculation

Table 1.9.2. shows the result of a calculation of a melting element of copper material. The restrictions are round holes of 1.5 mm in diameter and a distance of 1.8 mm from centre to centre in the line. The distange of the lines is 3 mm minimum. The cross section of all restrictions in a line is 1.00 mm^2 .

table 1.9.2.: (power factor 0.3 ; making angle 0 degrees)

current	4000	5000	6000	6500	7000	11000	21000	A
time until Ts	8.42	7.04	6.25	5.95	5.70	4.47	3.27	ms
integral value	191.7	172.6	162.6	159.1	156.2	142.8	129.9	*1000 A^2s
melting time	8.86	7.31	6.46	6.15	5.89	4.61	3.36	ms
melt. integral	219.4	196.0	184.5	180.7	177.3	162.1	147.9	*1000 A^2s
virt.melt.time	13.7	7.84	5.12	4.27	3.61	1.33	0.33	ms
run time	41.13	36.28	33.10	31.90	30.83	26.16	22.23	minutes

We did not take care to the influence of the resistance of the melting element to the current. When the test voltage is high enough then we may neglect this influence.



Literature:

1 D'Ans Lax: Taschenbuch für Chemiker u. Physiker , Springer 1967
2 Ulich-Jost: Physikalische Chemie
3 Robert M. Eisberg: mathem. Physik für Benutzer programm.Taschenrechner , Oldenbourg 1978

Conference papers , Trondheim 1984:
4 H.G.Rex: calculations of adiabatic models

Conference papers , Liverpool 1976:
5 P.H.McEwan and L.Warren: survey of num. methods for solving time varying fuse equations
6 M.M.McEwan and R.Wilkins: a decoupled method for predicting time-current charact.of fuses
7 A.Hirose: mathematical analysis of breaking performance of current limiting fuses

Session II

PRE-ARCING PHENOMENA 2

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