OPTIMUM CONDITIONS FOR TESTING ELECTRICAL FUSES AT MAXIMUM PREARCING ENERGY

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INTRODUCTION The occurrence of a serious fault in an electrical circuit energised by a power supply leads to the rapid accumulation of magnetic energy in the residual circuit inductance. If the supply is protected by a conventional fuse then this energy increases until such time that Ohmic heating in the solid fuse link is sufficient to cause melting. The link material subsequently constricts under the electromagnetic forces to form one or more arcs and the associated large increase in circuit resistance causes the fault current to fall rapidly to zero. The magnetic energy accumulated in the fault circuit before the fuse link disrupts is called the prearcing energy and it is subsequently dissipated in the circuit resistance as the current falls to zero during fuse disruption. For severe faults in a circuit protected by a quick-acting fuse the major resistance during this current cut-off phase is that associated with the disrupted fuse itself. However the current cut-off phase can be of sufficient duration for appreciable energy to be fed directly into the disrupted fuse from the power supply. Consequently the prearcing energy is usually the minimum energy that is dissipated in a fuse during the process of clearing a fault circuit. The duration of the cut-off phase is controlled by the magnitude of the fuse arc burning voltage in relation to the supply voltage. If this arc voltage is much greater than the opposing supply voltage then the current fall is very rapid and the total arc energy is equal to the prearcing energy. On the other hand if the arc voltage is equal to or less than the supply voltage, as for a fuse of insufficient voltage rating, then the current could rise after the prearcing phase, possibly leading to continuous arcing and failure to clear the circuit. The arc voltage is primarily determined by the fuse construction but is also considerably influenced by the circuit parameters during the cut-off phase. An increase in the length of the fuse link will increase the total arc voltage as will also its containment in a cartridge filled with a material that is efficient in arc quenching. The various functions performed by the filler material in the extinction process are discussed in a recent paper by Turner and Turner (1973). Baxter (1958) has measured the electrical

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energy dissipated in fuses undergoing short-circuit tests and has shown conclusively that the prearcing energy is approached as the length of the fuse link is increased for a given filler material and supply voltage.

The ability of a fuse to clear a fault circuit without the cartridge exploding or splitting is primarily determined by the energy dissipated within it. Each category of fuse manufactured is put through a short-circuit test to verify that it will safely contain the maximum energy likely to be encountered in the circuit it is intended to protect. Such testing is an iterative and therefore a costly procedure unless the test circuit parameters can be optimised beforehand so as to dissipate maximum energy within the fuse under test. The calculations here determine the fault circuit parameters required to maximise the prearcing energy and are therefore directly applicable to the testing of fuses exhibiting rapid cut-off. They also serve as convenient initial conditions for testing fuses for which the arc voltage is not significantly greater than the supply voltage. For D.C. power supplies Wheeler (1972) has located a maximum in the prearcing energy for a given fuse in a fault circuit of specified time constant and his analysis enables the optimum fault current and prearcing time to be evaluated. Consideration of A.C. power supplies introduces another variable into the fault circuit, namely the phase in the supply voltage cycle when the fault is applied, referred to as the closing angle. Boehne (1946) and Toniolo and Cantarella (1969) have studied the prearcing phase in circuits of power factor of 0 and 0.2 respectively. Their results enable the prearcing energy for a given fuse to be evaluated as a function of closing angle and fault current, however the possibility of a maximum is not considered. Wheeler (1973) has evaluated the maximum energy and associated cut-off currents for fault circuits of various power factors, all at zero closing angle. He points out that, for a given power factor, an overall maximum energy must exist for some optimum closing angle. Recently Wilkins and McEwan (1975a) evaluated this overall energy maximum and the associated optimum fault currents for circuits of selected power factors less than 0.5. This paper determines the overall maximum prearcing energy for all power factors together with the optimum values of fault current, closing angle, prearcing time and cut-off current. These parameters are conveniently expressed in units that enable application to any supply circuit and almost any fuse.

<u>FAULT CURRENT WAVEFORM</u> The fault circuit is characterised by its inductance L, and resistance R, however for fuse applications it is standard procedure to define the circuit by one parameter, namely by the time constant $\tau = L/R$ for the D.C. case and by the power factor $\cos\alpha = R/(R^2+\omega^2L^2)^{1/2}$ for the A.C. case, where ω is the angular frequency of the supply. During the prearcing phase the fuse link is transformed from the solid to the liquid state, necessitating an increase in fuse resistance. The change in total circuit resistance resulting from this process is usually very small and in the following analysis it is assumed that the

parameters R and L do not vary during this phase. The fault current is characterised by the prospective current I_0, which is the amplitude of the current that would flow in the steady-state if the fuse link were replaced by a perfect short-circuit. If V denotes the D.C. supply voltage or the amplitude of the A.C. supply voltage then

$$\text{D.C.} \quad I_0 = V/R, \qquad \text{A.C.} \quad I_0 = V/(R^2+\omega^2L^2)^{1/2} \tag{1}$$

The time dependence of the fault current $I(t)$ can then be expressed in terms of I_0 as follows

$$\text{D.C.} \quad I(t)/I_0 = 1 - \exp(-t/\tau), \qquad \text{A.C.} \quad I(t)/I_0 = \sin(\omega t+\beta-\alpha) - \sin(\beta-\alpha)\exp(-\omega t\cot\alpha), \tag{2}$$

where β is the phase angle in the supply voltage cycle when the fault occurs and time is measured from the occurrence of this fault. Figure 1 shows the current waveforms for power factors of 0.9 and 0.1 with closing angles β varying between 0° and 180°. For the particular case of $\beta=\alpha$ equation (2) shows that the transient term is zero and the peak fault current therefore equal to the prospective current. This is termed the case of symmetrical current and the closing angles $\beta=\pi/6$ for $\cos\alpha=0.9$ and $\beta=\pi/2$ for $\cos\alpha=0.1$ in Figure 1 are quite close to this situation. In general the approach to the steady-state is rapid for large power factors and slow for small power factors as required by equation (2) since the transient term becomes negligible for times such that $\omega t \gg \tan\alpha$. Also, for small power factors, the peak fault current in its initial stages can exceed the prospective current by up to a factor of two.

PREARCING TIME The fault current waveform is maintained until such time the sufficient Joule heating has taken place in the fuse link to disrupt it. If the heating takes place on a time scale that is much shorter than that required for significant heat loss from the link then the total Joule energy input up to the time of disruption can be equated to the thermal capacity of the link at its state of disruption.

$$\int_0^{t_p} J^2(t)\,dt = K. \tag{3}$$

$J(t)$ is the current density, t_p is the prearcing time and K a parameter that depends only on the physical properties of the link metal. Uniform current density is assumed, implying that the time-scale of the process is sufficiently long for skin effects to be negligible. In almost all applications of quick-acting fuses to conventional power systems the prearcing time falls within the upper and lower limits imposed by the above considerations. Morgan (1971) has given a very thorough account of the calculation of K for various metals following the technique proposed by Gibson (1941). Measurements of prearcing time for copper fuse links in power circuits by Baxter (1950) and Wheeler (1972) show that about 30% of the link metal must be transformed to the liquid state before current cut-off occurs. This condition requires the value $K = 9.5 \times 10^8\ A^2s\ cm^{-4}$ for copper and $K = 6.1 \times 10^8\ A^2s\ cm^{-4}$ for silver. If the link is of uniform cross-sectional area a then $J(t) = I(t)/a$

and equation (3) can be written in the following form

$$\text{D.C.} \quad I_o^2 = K\alpha^2/\tau \Big/ \int_0^{t_p/\tau} [I(t)/I_o]^2 \, d(t/\tau),$$

$$\text{A.C.} \quad I_o^2 = K\alpha^2\omega \Big/ \int_0^{\omega t_p} [I(t)/I_o]^2 \, d(\omega t). \qquad (4)$$

Reference to equation (2) shows that, for a given fault circuit, the integrals here are only a function of t_p, τ or of t_p, α, β; implying that equation (4) is essentially a relation between prearcing time and prospective current. There is one type of fuse link, the notched link, that cannot be treated in this simple manner. For such a link the length of the constricted portion may not be sufficiently great for the neglect of longitudinal heat losses during the prearcing phase. This problem has recently been treated by Wilkins and McEwan (1975b).

PREARCING ENERGY The prearcing energy is the circuit inductive energy at the commencement of link disruption,

$$W = \tfrac{1}{2} L I^2(t_p). \qquad (5)$$

If the inductance is expressed in terms of the circuit time constant or power factor then equations (1), (4) and (5) can be combined to give

$$\text{D.C.} \quad W/(V^2 K\alpha^2 \tau)^{1/2} = [I(t_p)/I_o]^2 \Big/ 2 \Big[\int_0^{t_p/\tau} [I(t)/I_o]^2 \, d(t/\tau)\Big]^{1/2},$$

$$\text{A.C.} \quad W/(V^2 K\alpha^2/\omega)^{1/2} = [I(t_p)/I_o]^2 \sin\alpha \Big/ 2 \Big[\int_0^{\omega t_p} [I(t)/I_o]^2 \, d(\omega t)\Big]^{1/2}. \qquad (6)$$

Reference to equation (2) shows that the right-hand-side of this equation is a function only of t_p and τ or of t_p, α and β. This means that the energy can be expressed either as a function of the prearcing time, or of the cut-off current using equation (2), or of the prospective current from equation (4). Figure 2 shows the D.C. prearcing energy expressed as a function of these three variables. The maximum energy and its optimum coordinates are

$$W = 0{\cdot}488 (V^2 K\alpha^2 \tau)^{1/2}, \qquad t_p = 0{\cdot}943\tau,$$
$$I = 0{\cdot}611\, I_o, \qquad I_o = 2{\cdot}62\, (K\alpha^2/\tau)^{1/2}.$$

This maximum and the optimum current cut-off ratio have previously been evaluated by Wheeler (1972). Figures 3 and 4 show the A.C. prearcing energy expressed as a function of current cut-off ratio for the current waveforms depicted in Figure 1. Only the first current loops are presented and it is apparent that there is an overall maximum energy for a closing angle near $\beta = \pi/3$ for $\cos\alpha = 0{\cdot}9$ and in the vicinity of $\beta = 0$ for $\cos\alpha = 0{\cdot}1$. These energies have been evaluated for all power factors by maximising equation (6) with respect to ωt and β simultaneously. Figure 5 shows the energy maxima together with the optimum closing angles and Figure 6 shows the optimum prearcing times together with the current cut-off ratios. The sum of the closing angle and the prearcing time is termed the arcing angle, which is simply the phase in the supply voltage waveform when cut-off begins. This angle is shown in Figure 7 together with the optimum prospective currents.

DISCUSSION The manner in which the supply voltage V and the fuse parameter Ka^2 enter into these calculations is interesting. Figures 2,5,6 and 7 show that V enters only into the prearcing energy whereas Ka^2 enters only into the energy and the prospective current. The remaining optimised parameters, namely the prearcing time ratio (t_p/τ or ωt_p), the closing angle and the current cut-off ratio, have values that are either fixed for the D.C. case or are functions only of power factor for the A.C. case. For given values of V and Ka^2 the important parameters for carrying out an A.C. short-circuit fuse test are the optimum closing angle of Figure 5 and the optimum prospective current of Figure 7. The prospective current varies smoothly between $0{\cdot}688(Ka^2\omega)^{1/2}$ for $\cos\alpha = 0$ and infinity for $\cos\alpha = 1$. However the closing angle shows an abrupt behaviour around $\cos\alpha = 0{\cdot}4$ where it departs from a constant value of zero and subsequently reaches $\pi/2$ at $\cos\alpha = 1$. The optimum prearcing time in Figure 6 exhibits a similar, less pronounced behaviour around $\cos\alpha = 0{\cdot}4$ but there is no analogous behaviour in the arcing angle of Figure 7 that is formed by summing these two quantities. This optimum arcing angle varies between $\pi/2$ and $2{\cdot}44$ radian, indicative of cut-off between circuit voltage maximum and 64% of this maximum on the falling voltage characteristic.

It must be emphasised that the prearcing energy only represents the total fuse energy dissipation if the supply voltage is very much less than the arc burning voltage. Under such conditions fuse testing need only be carried out in a D.C. circuit since the A.C. performance can then be assessed from the figures presented here.

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FIGURE CAPTIONS Figure 1. Current waveforms for power factors of 0.1, 0.9 and for various closing angles β (radian) within the voltage cycle.

Figure 2. Prearcing energy W for a D.C. circuit of time constant τ as a function of cut-off current I, prearcing time t_p and prospective current I_o.

Figure 3. Prearcing energy W for a circuit of power factor 0.9 and prospective current I_o as a function of cut-off current I and for various closing angles β (radian).

Figure 4. Prearcing energy W for a circuit of power factor 0.1 and prospective current I_o as a function of cut-off current I and for various closing angles β (radian).

Figure 5. Maximum prearcing energy W and optimum closing angle β (radian) as a function of circuit power factor $\cos\alpha$.

Figure 6. Optimum prearcing time t_p (ωt_p in radian) and optimum cut-off current I as a function of circuit power factor $\cos\alpha$.

Figure 7. Optimum prospective current I_o and optimum arcing angle $\omega t_p+\beta$ (radian) as a function of power factor $\cos\alpha$.

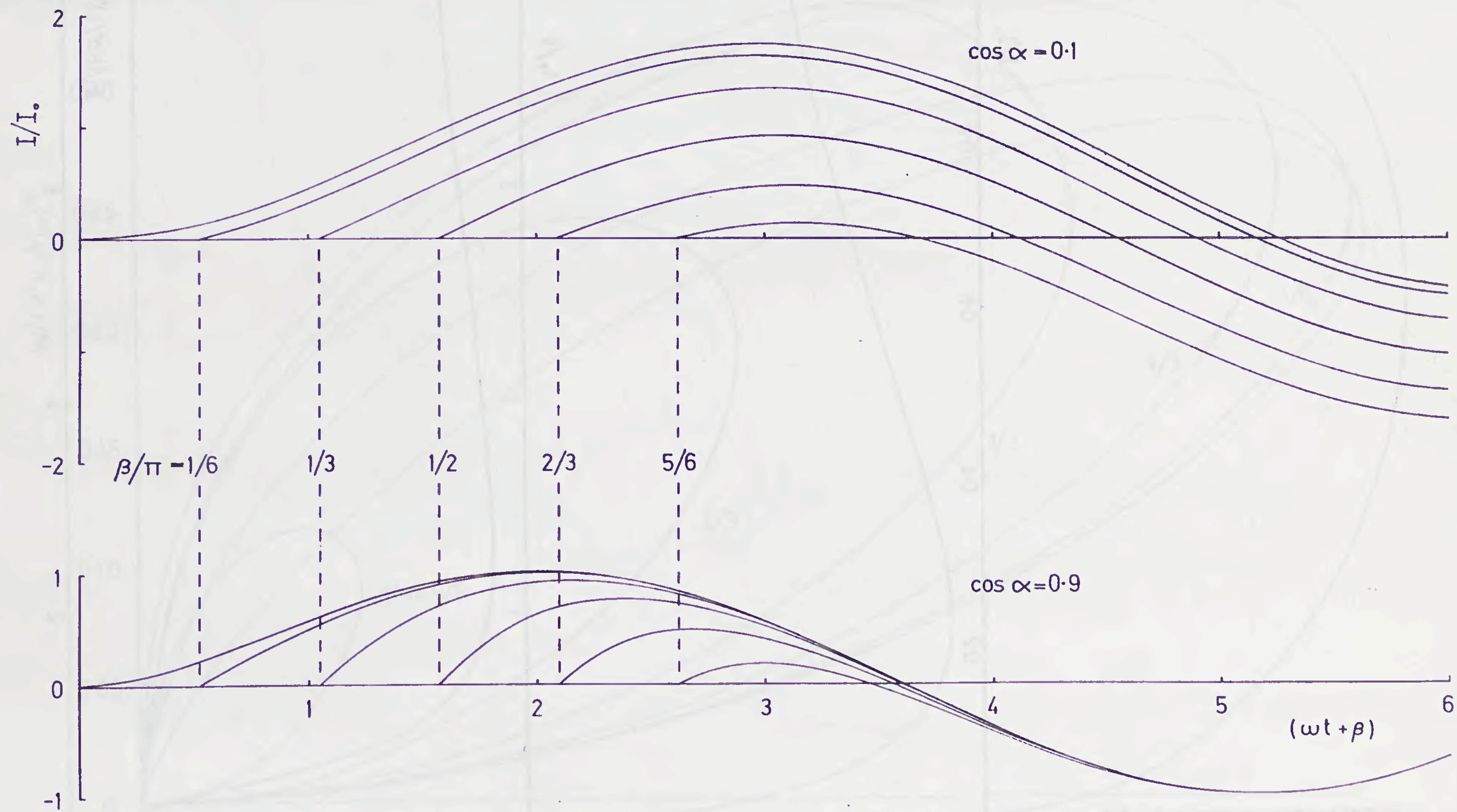


Figure 1.

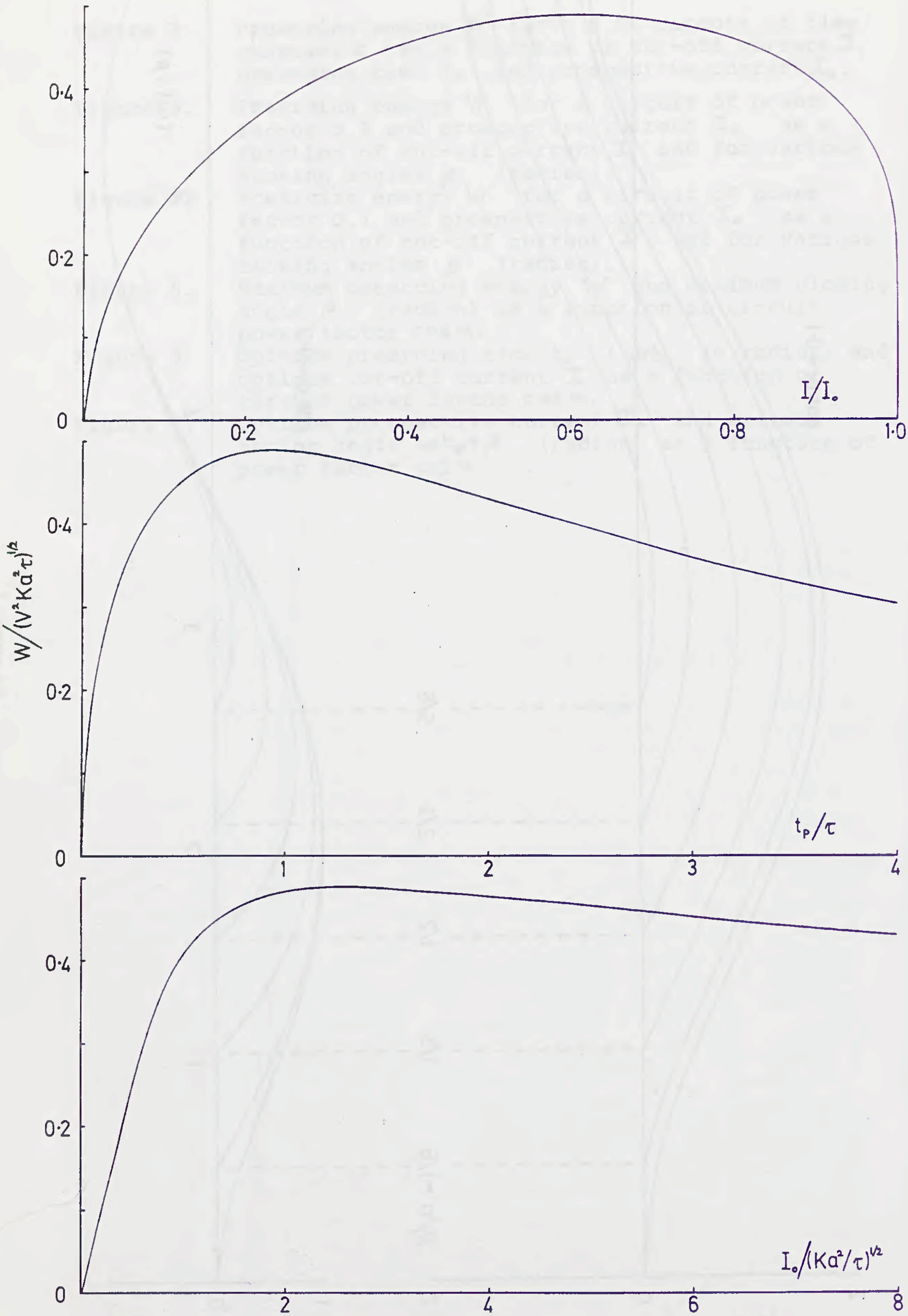
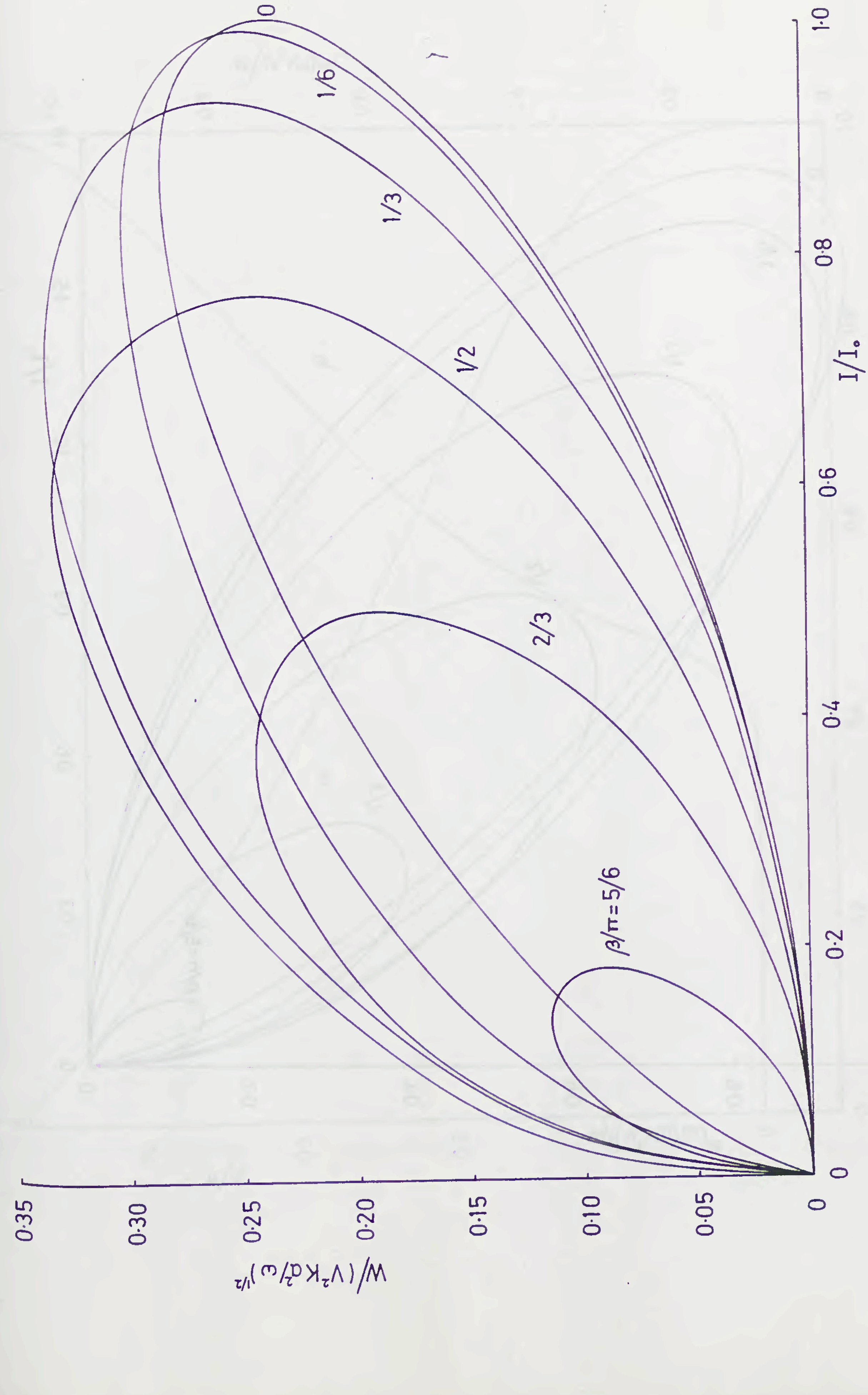


Figure 2



0
1/6
1/3
1/2
2/3
β/π=5/6
0
0·05
0·10
0·15
0·20
0·25
0·30
0·35
W/(V²Ka²/ω)^{1/2}
0
0·2
0·4
0·6
0·8
1·0
I/I₀

Figure 3.

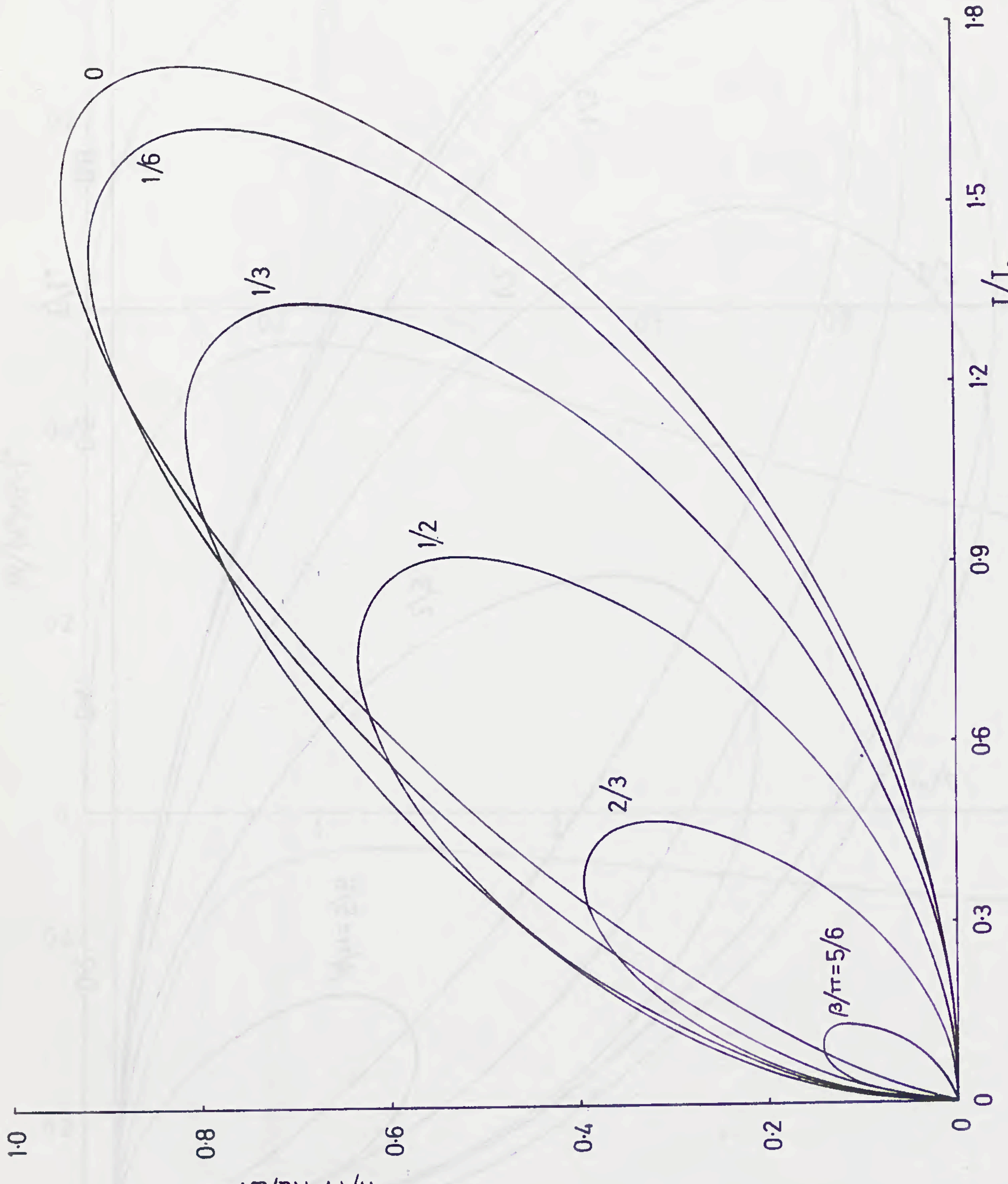


Figure 4.

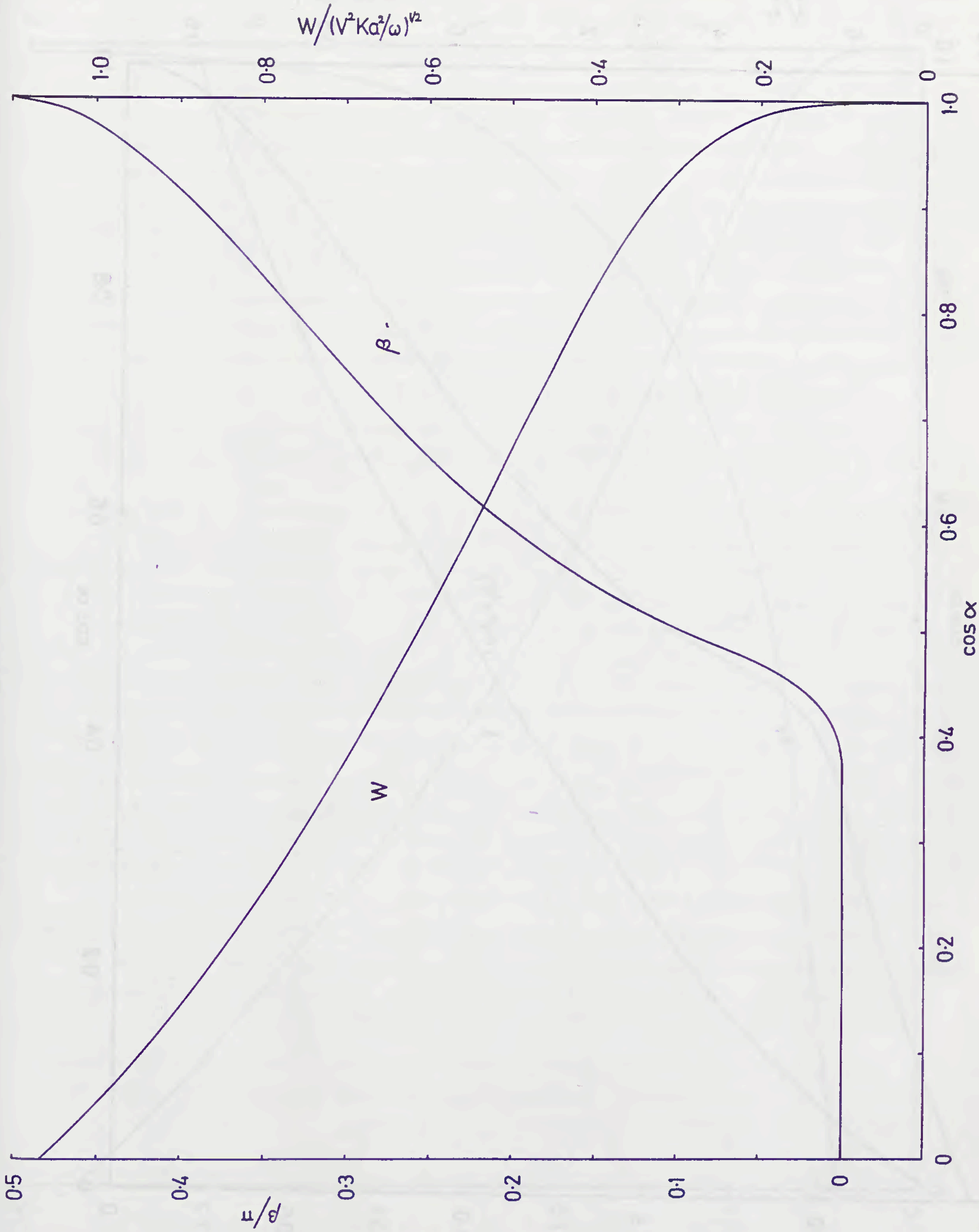


Figure 5.

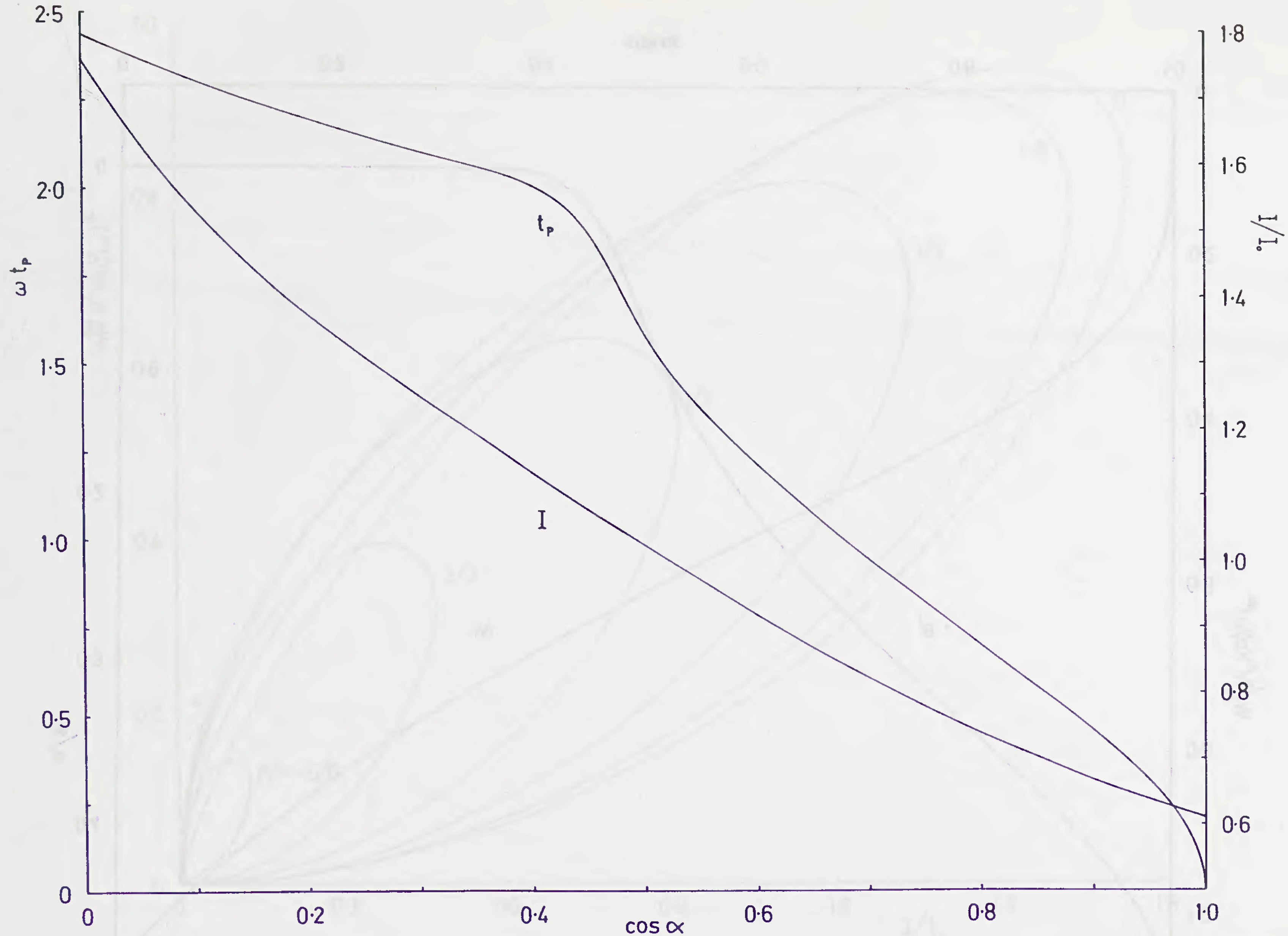

2·5
2·0
1·5
1·0
0·5
0
$\frac{t_p}{3}$
1·8
1·6
1·4
1·2
1·0
0·8
0·6
I/I_o
t_p
I
0
0·2
0·4
0·6
0·8
1·0
cos α

Figure 6.

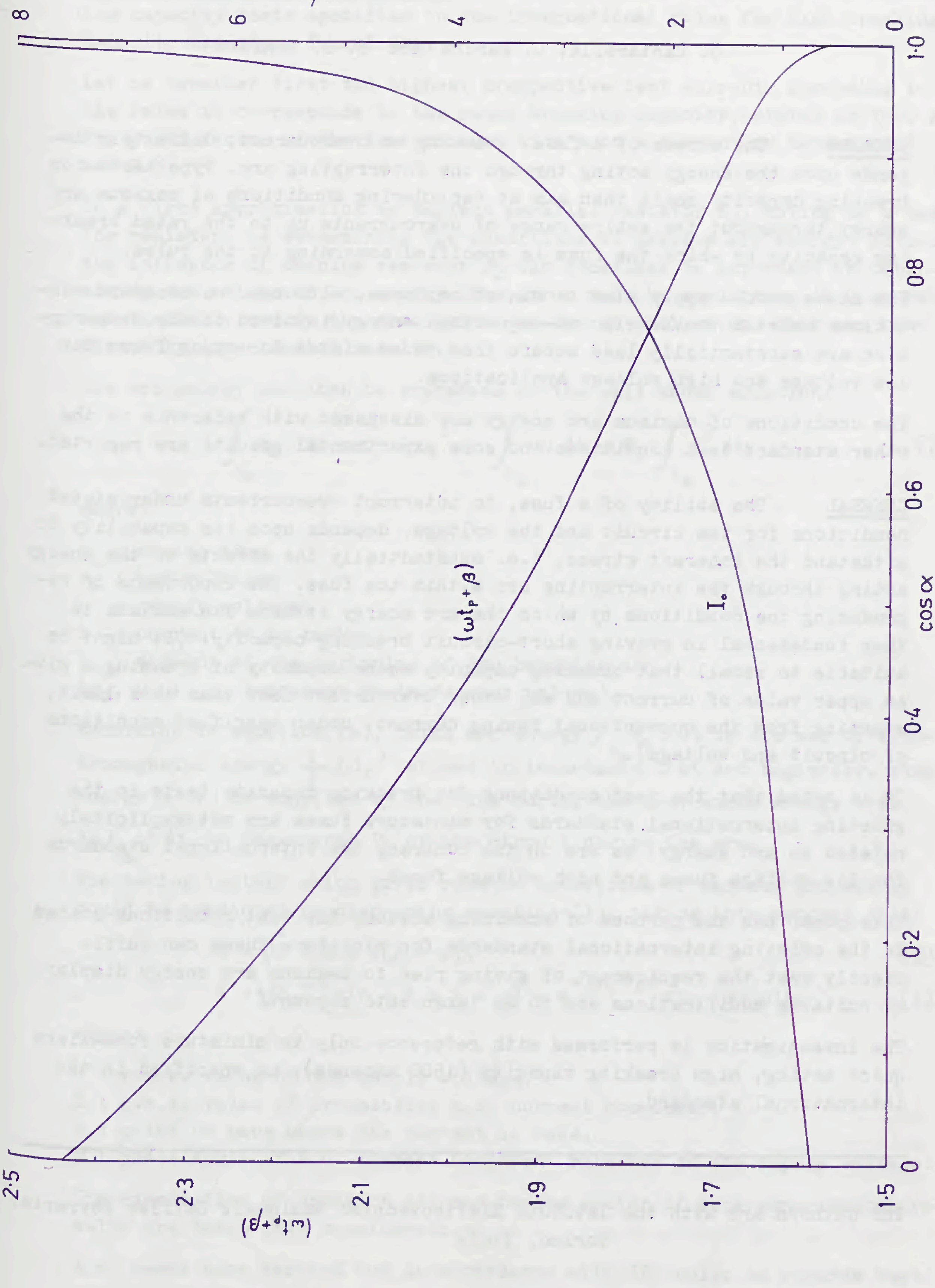

I₀/(Ka²ω)^1/2
8
6
4
2
0
1·0
0·8
0·6
0·4
0·2
0
(ωt_p+β)
I₀
cos α
2·5
2·3
2·1
1·9
1·7
1·5
(ωt_p+β)

Figure 7